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Color superconductor with a color-sextet condensate

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Abstract. We analyze color superconductivity of one massive flavor quark matter at moderate baryon density with a spin-zero color-sextet condensate. The most general form of the order parameter implies complete breakdown of the $SU(3) \times U(1)$ symmetry. However, both the conventional fourth-order polynomial effective bosonic description and the fermionic NJL-type model in the mean-field approximation favor an enhanced $O(3)$ symmetry of the ground state. We suggest two mechanisms how the complete symmetry breakdown could be achieved.

1 Introduction

Color superconductivity is by now a widely accepted candidate for the physics of cold dense deconfined quark matter [1]. This belief is based on a simple observation that the QCD-induced force between two quarks may be attractive, for a suitable combination of their color charges and due to its non-abelian nature. Such an attraction is then supposed to give rise to bound (Cooper) pairs of quarks, very much like in ordinary superconductors bound electron pairs form due to a phonon-mediated effective interaction.

In this contribution we wish to present a simple model for color superconductivity which favors pairing of quarks of a single flavor in a color-symmetric (sextet) channel, unlike the interactions usually used in literature. First, in the next two sections, we review briefly the framework of color superconductivity and give some motivation for our further considerations. Then, we move on to a description of our model, starting from general kinematical arguments, and proceeding with a semi-microscopic treatment within a model of a Nambu–Jona-Lasinio (NJL) type [2]. We end with pointing out some interesting features of our model and with prospects for future work.

2 Framework

The behavior of cold dense quark matter is described by QCD which is believed to be the fundamental theory of strong interactions. At very high density the quark Fermi energy is very large and due to asymptotic freedom of QCD we can use weak coupling methods to do calculations from first principles. Thus it can be rather rigorously shown that the QCD interaction indeed leads to formation of quark Cooper pairs and hence to color superconductivity. The ground state of very dense quark matter is expected to be the so-called Color-Flavor-Locking (CFL) phase [3].

Unfortunately, current QCD calculations are only reliable at very high density, corresponding to quark chemical potential larger than 10^8 MeV [4]. This is far too much to be of any use for neutron star phenomenology, which is so far the only known possible experimental signature of color superconductivity. Rather, chemical potentials not larger than a few hundreds MeV are relevant there. This corresponds to the strong-coupling, non-perturbative regime of QCD.

It is widely believed that superconductivity is relevant also in this strong-coupling regime of QCD. For this purpose, various types of interactions are used to model the QCD-induced attraction between quarks. As far as we assume that quark matter can be described by a purely fermionic effective Lagrangian (gluons having been integrated out), renormalization group (RG) arguments suggest that we turn our attention to contact four-quark interactions of NJL type. On the other hand, RG itself does not tell us which particular interaction to choose. We are therefore in a position to guess what the effective interaction might be and provide some motivation for our choice. This is so far the best we can do.

3 Motivation

There are various standard four-quark interactions that are widely use in literature [1]. They take over their color and flavor structure from both perturbative (one gluon exchange) and non-perturbative (instanton-mediated interaction) physics. We emphasize, however, that the corresponding coupling constants can only be explicitly calculated when the QCD coupling is weak, and therefore we regard them as particular examples of a 'good guess' mentioned above.

These 'standard' interactions have the common feature that they favor pairing of quarks in the color-antisymmetric (antitriplet) channel. We would like to propose here a new kind of effective interaction that, on the contrary, favors pairing in the color-sextet channel.

Our motivation goes back twenty years to a work by Hansson et al. [5], who investigated the QCD-induced force between two gluons. Although the main aim of these authors was to interpret the QCD vacuum as a condensate of colorless spin-zero glueballs, they classified also the colored two-gluon states according to attractiveness or repulsiveness of the QCD force. They found that, apart from the above mentioned colorless glueball, the most strongly bound state is that of spin zero, which transforms as a color octet.

Even though such a state certainly does not exist as an excitation of the QCD vacuum, due to color confinement, it might be a relevant degree of freedom in the dense, deconfined phase. We observe that, if present, this mode would certainly interact with quarks and its exchange would lead to an effective interaction of the form

$$\mathcal{L}_{\text{int}} = G(\bar{\psi}\lambda\psi)^2, \tag{1}$$

where $G > 0$ is a theoretically unspecified coupling constant.

This interaction is attractive in the color-sextet channel, and might perhaps compete with the standard interactions in the pairing of a single flavor for the following reason. In the region of moderate quark densities where the s quark mass is too large compared with the chemical potential for the CFL phase to be energetically favorable, the u and d quarks will pair together to form the so-called 2SC phase. The s quark cannot pair with u and d, but it can pair with itself. This is the $2+1$ flavor scheme [1].

Now the standard interactions favor pairing of two s quarks in a color-antitriplet state of spin one. These pairs are, however, only weakly bound, with the gap in the spectrum estimated to be of order of tens or a hundred keV, while in the 2SC phase the gap is roughly tens or a hundred MeV [1].

To conclude, if the interaction (1) should be able to compete with the standard ones at all, it would probably be in the pairing of a single flavor, namely the s quarks.

4 Color-sextet superconductor

With the motivation just given, we consider quark matter of one massive flavor and investigate its behavior under the effective interaction (1). Here we just sketch the main ideas, for details of calculations see [6].

4.1 Kinematics

As stressed before, the interaction (1) favors pairing in the color-sextet channel which means, by the Pauli principle, spin zero of the quark pairs. We therefore expect that the system develops a condensate

$$\langle \psi_i^T (C\gamma_5)\psi_j \rangle \propto \phi_{ij}, \tag{2}$$

where ϕ_{ij} is a complex, symmetric 3×3 matrix.

By exploiting the underlying $SU(3)\times U(1)$ symmetry (the $SU(3)$ being the color gauge group, and the global $U(1)$ representing the quark number conservation), the matrix ϕ can be brought to a real, non-negative diagonal form, $\Delta = \text{diag}(\Delta_1, \Delta_2, \Delta_3)$. Therefore, the phase is characterized by three independent order parameters. As we will see, the properties of the phase strongly depend on relative values of the Δ's.

4.2 Symmetry-breaking patterns

In the presence of the condensate (2), the underlying symmetry is necessarily broken. The details of the symmetry-breaking pattern, however, strongly depend on the particular choice of the order parameters. Here we do not list all the possibilities (see [6] for that purpose), but rather mention two of them that will be of interest for us in the following.

First, if all the Δ's are non-zero and different, the continuous $SU(3)\times U(1)$ symmetry is completely broken. As a result, all eight gluons acquire non-zero masses via the Higgs mechanism, and there is one physical Goldstone boson corresponding to the broken global $U(1)$.

Second, if all the Δ's are non-zero but equal, there is an unbroken $O(3)$ symmetry left. Three gluons stay massless, whereas the remaining five receive equal masses, as the breaking $SU(3)\to SO(3)$ is isotropic.

4.3 Effective bosonic description

For a phenomenological parameterization of the sextet phase, we describe it first à la Ginzburg and Landau. We thus imagine ϕ as a vacuum expectation value of a composite scalar field Φ which we use to construct an effective $SU(3)\times U(1)$-invariant Lagrangian. The condensate ϕ is then given by the minimum of the effective potential $V(\Phi)$.

The most general potential $V(\Phi)$ can be constructed from three algebraically independent invariants $\det(\Phi^\dagger\Phi)$, $\text{tr}(\Phi^\dagger\Phi)$, and $\text{tr}(\Phi^\dagger\Phi)^2$, whose values at the minimum in turn uniquely specify the three order parameters Δ_1, Δ_2, Δ_3.

To obtain some concrete prediction, we try the most general quartic potential (which is the usual starting point for a Ginzburg–Landau analysis)

$$V_{\text{quart}}(\Phi) = -a\,\text{tr}\,\Phi^\dagger\Phi + b\,\text{tr}(\Phi^\dagger\Phi)^2 + c(\text{tr}\,\Phi^\dagger\Phi)^2. \tag{3}$$

Minimization of V_{quart} suggests that ϕ be proportional to the unit matrix i.e., $\Delta_1 = \Delta_2 = \Delta_3$. We should, however, be careful as the invariant $\det(\Phi^\dagger\Phi)$ is missing in (3) and so V_{quart} contains only incomplete information about the phase. We will see next how a similar lack of information is manifested in a fermionic description.

4.4 NJL-type fermionic description

We now turn to a semi-microscopic description in terms of quark fields interacting via the effective interaction (1).

Our model Lagrangian for one massive quark flavor at non-zero density (represented here by the chemical potential, μ) reads

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m + \mu\gamma_0)\psi + G(\bar{\psi}\lambda\psi)^2. \tag{4}$$

We treat the Lagrangian (4) in the mean-field approximation. We use Fierz identities to rearrange the quark fields in the interaction term and introduce the diquark condensate[1]

$$\langle \psi_i^T (C\gamma_5)\psi_j \rangle = \frac{3}{2G}\Delta_i \delta_{ij}. \tag{5}$$

The mean-field Lagrangian becomes

$$\mathcal{L}_{\text{m-f}} = \bar{\psi}(i\partial\!\!\!/ - m + \mu\gamma_0)\psi + \frac{1}{2}\bar{\psi}\Delta(C\gamma_5)\bar{\psi}^T - \frac{1}{2}\psi^T\Delta^\dagger(C\gamma_5)\psi. \tag{6}$$

Now $\mathcal{L}_{\text{m-f}}$ is already bilinear in the fields, so the spectrum of the corresponding Hamiltonian can be calculated exactly. We find that there are two kinds of excitations above the ground state, one quark-like and one antiquark-like. Their dispersion relations are

$$E_\pm^i(\mathbf{p}) = \sqrt{\left(\sqrt{|\mathbf{p}|^2 + m^2} \pm \mu\right)^2 + |\Delta_i|^2}, \tag{7}$$

where $i = 1, 2, 3$ stands for the three colors and the gaps Δ_i are determined as solutions to the gap equation which, at non-zero temperature T reads

$$1 = \frac{2}{3}G\int^\Lambda \frac{d^3\mathbf{p}}{(2\pi)^3}\left(\frac{1}{E_+^i}\tanh\frac{E_+^i}{2T} + \frac{1}{E_-^i}\tanh\frac{E_-^i}{2T}\right). \tag{8}$$

5 Conclusions

Let us look at what we have obtained. First of all, we stress the fact that the gap equation is generally a matrix equation for the gap matrix Δ which, as we have explained above, can be sought in the form of a real diagonal matrix with non-negative entries. As we clearly see, in the mean-field approximation, its matrix structure is trivial – Equation (8) represents three identical equations for the three gaps Δ_i, and all of them thus acquire the same value (due to the monotonicity of the integrand, (8) can apparently have only one non-zero solution).

Thus the mean-field approximation suggests that the symmetry is broken down to $O(3)$. There is, however, no general argument that forces the dynamics of the system to preserve the $O(3)$ subgroup of the $SU(3) \times U(1)$ when, by symmetry considerations only, it can be completely broken (see discussion in Sect. 4.2). We therefore suspect that this result is just an artifact of the mean-field approximation. There is a simple physical argument that supports this suspicion. In the mean-field approximation, each quark moves in an effective mean field generated by all the quarks, thus reducing the problem to a single-particle one. Now quarks of a single color, say blue, generate a mean field which is felt again only by blue quarks, and analogously for red and green colors. Hence the three colors do not mix at all and there is no way how differences between the Δ's could arise.

In full theory, however, the three colors mix non-trivially and this could possibly lead to asymmetric solutions of the gap equation. Another mechanism that could serve for that purpose is to add the two light quark flavors pairing in the color-antitriplet channel, as in the standard 2SC phase. But the transformations of the antitriplet and sextet condensates are locked, and thus both condensates cannot be brought simultaneously to their standard forms. In this case, it is even not apparent how many independent order parameters there are.

Finally, it would be nice to know whether there really is the desired color-octet spin-zero mode in the dense deconfined QCD matter. Then, we should try to estimate the value of the effective coupling G so that we can assess whether the interaction (1) might compete with the interactions standard in literature.

All these issues will be the subject of our future work.

[1] We also, in this simple model, neglect all other possible condensates which would have to be included in a more careful self-consistent analysis.

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References

1. K. Rajagopal and F. Wilczek: *The condensed matter physics of QCD*, `hep-ph/0011333`
2. S.P. Klevansky: "The Nambu–Jona-Lasinio model of quantum chromodynamics", Rev. Mod. Phys. **64**, 649–708 (1992)
3. M.G. Alford, K. Rajagopal, and F. Wilczek: "Color-flavor locking and chiral symmetry breaking in high density QCD", Nucl. Phys. B **537**, 443–458, (1999) [`hep-ph/9804403`]
4. K. Rajagopal and E. Shuster: "On the applicability of weak-coupling results in high density QCD", Phys. Rev. D **62** 085007 (2000), [`hep-ph/0004074`]
5. T.H. Hansson, K. Johnson, and C. Peterson: "The QCD vacuum as a glueball condensate", Phys. Rev. D **26**, 2069 (1982)
6. T. Brauner, J. Hošek, and R. Sýkora: "Color superconductor with a color-sextet condensate", `hep-ph/0303230`. To be published in Phys. Rev. D